MIT. 7238

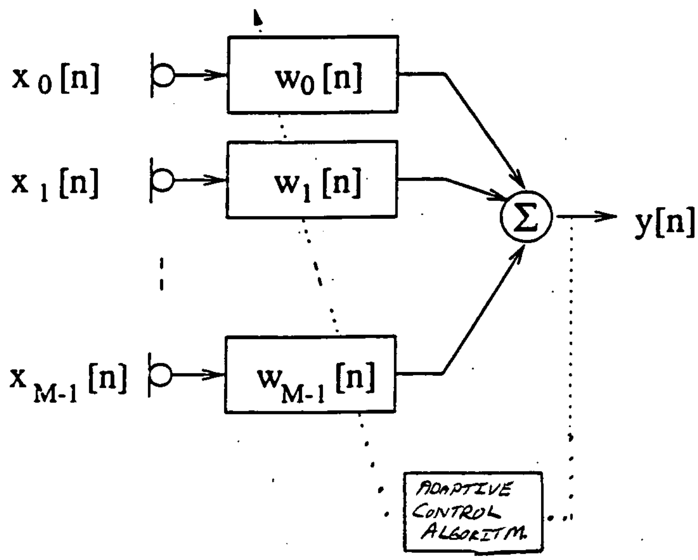


FIG. 1

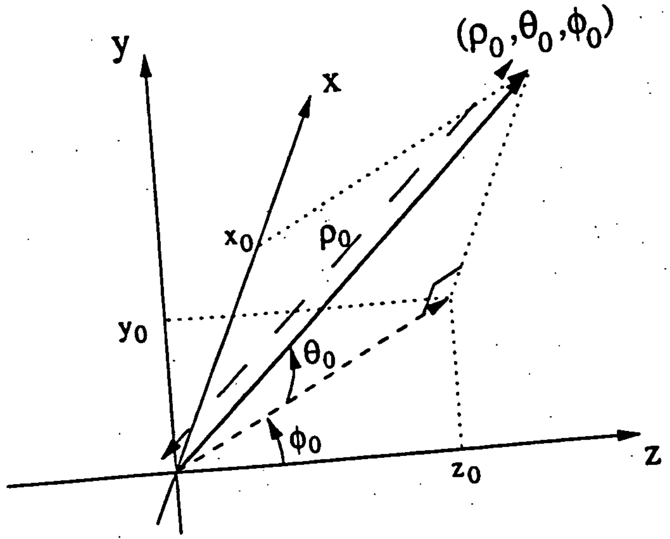
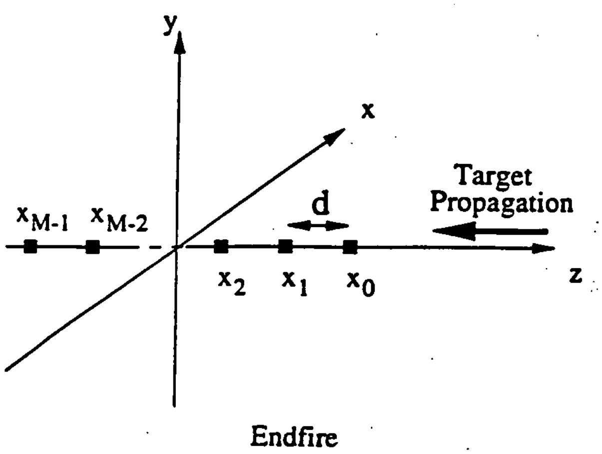
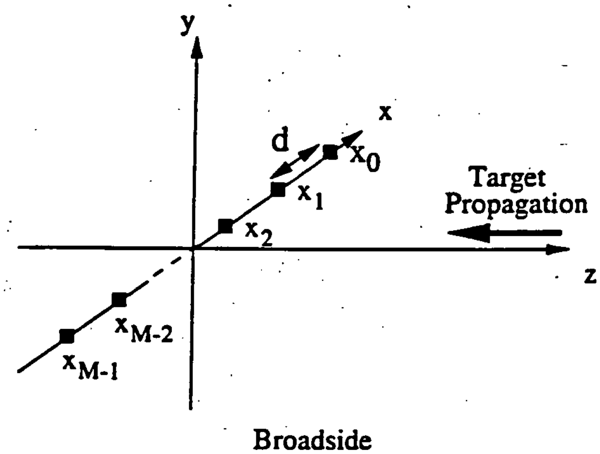


FIG 2



y
x
Target
Propagation
d
x_{M-1}
x_{M-2}
x_2
x_1
x_0
z
Endfire

FIG. 3A



y
x
d
x_0
x_1
x_2
Target
Propagation
z
x_{M-2}
x_{M-1}
Broadside

FIG. 3B

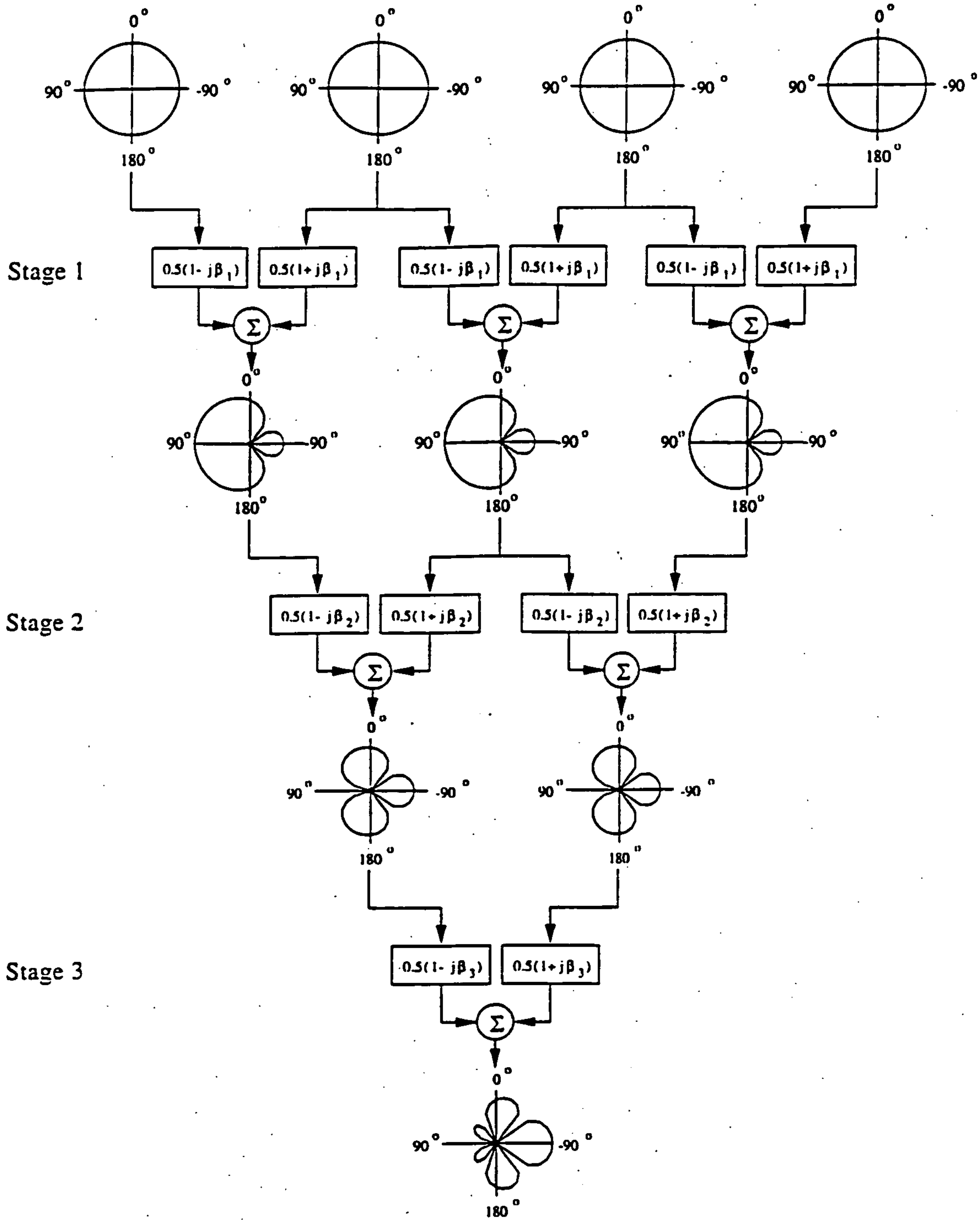

0°
90°
-90°
180°
Stage 1
0.5(1- jβ1)
0.5(1+jβ1)
Σ
Stage 2
0.5(1- jβ2)
0.5(1+jβ2)
Stage 3
0.5(1- jβ3)
0.5(1+jβ3)

FIG. 4

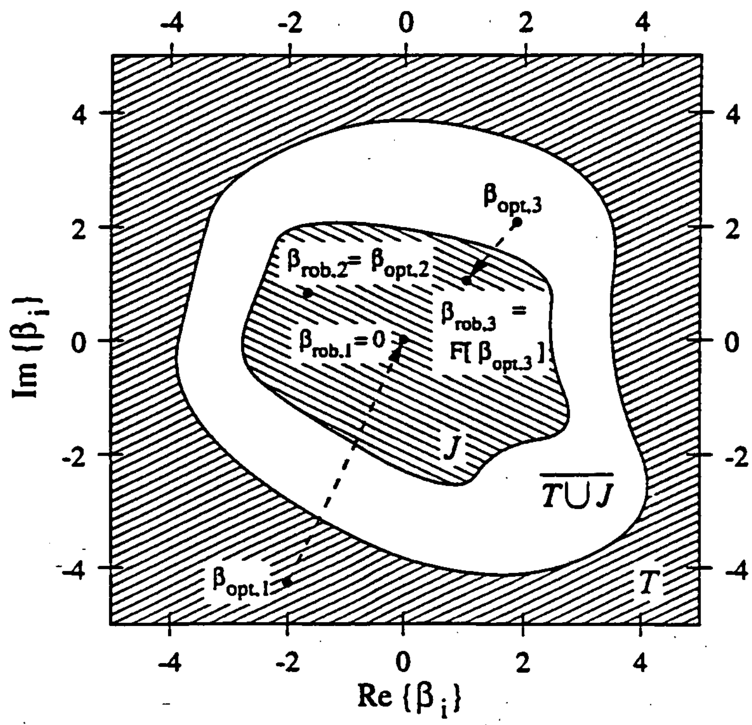

-4
-2
0
2
4
Im {β i}
Re {β i}
$\beta_{opt,3}$
$\beta_{rob,2} = \beta_{opt,2}$
$\beta_{rob,3}$ =
$F[\beta_{opt,3}]$
$\beta_{rob,1} = 0$
J
$\overline{T \cup J}$
T
$\beta_{opt,1}$

FIG. 5

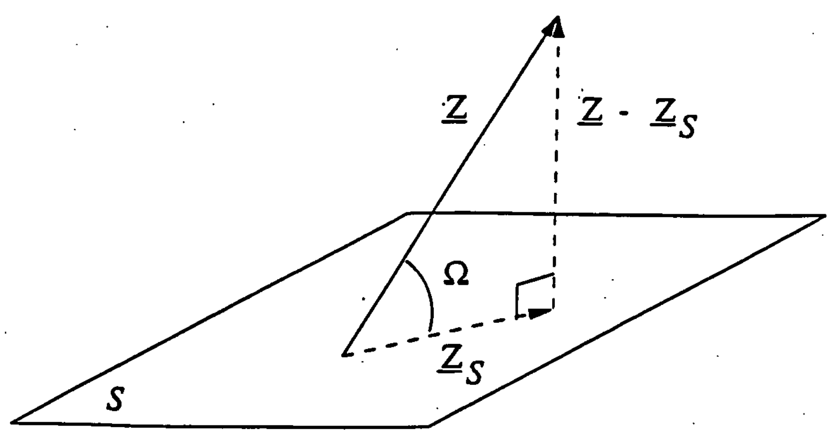
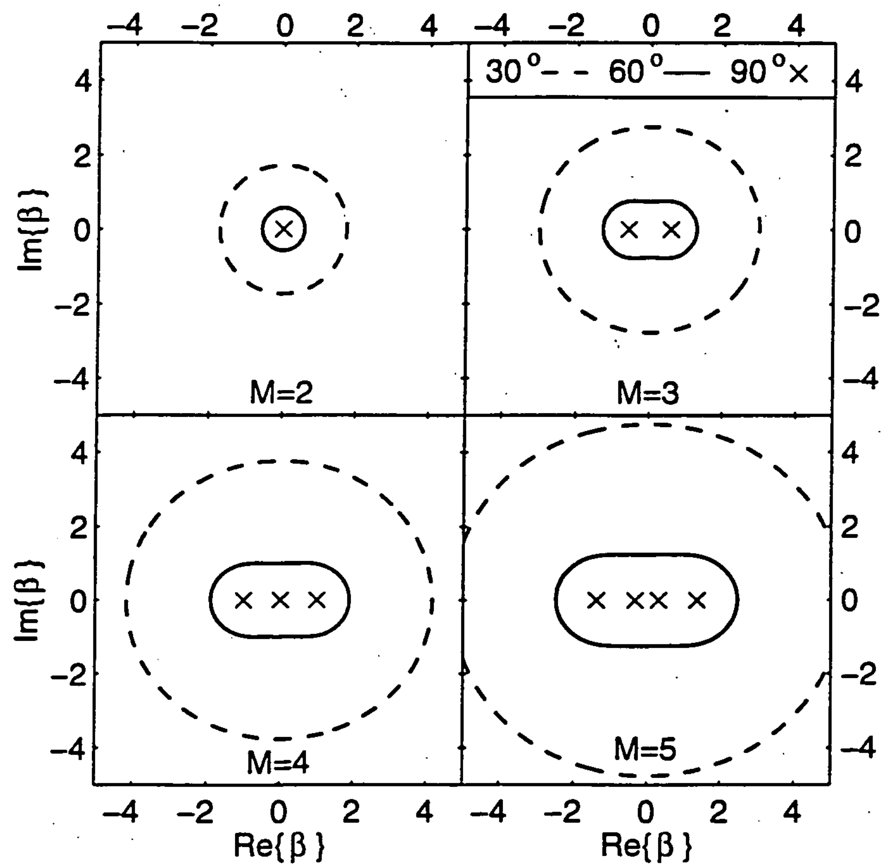


FIG. 6



Contours for $\Omega_i(\underline{\beta}) = 30°$ (- - -) $60°$ (—), and $90°$ (x) for $M = 2, 3, 4,$ and 5 element arrays.

FIG. 7

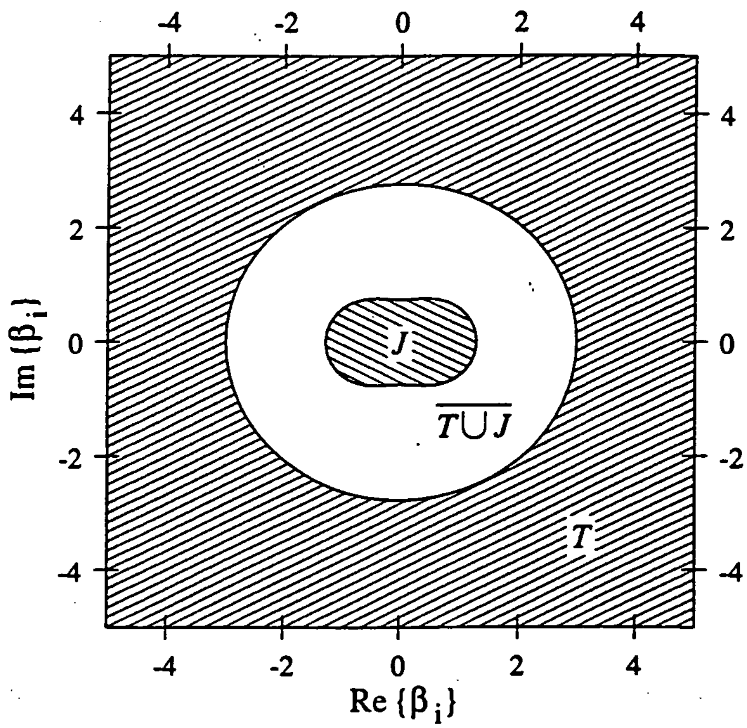
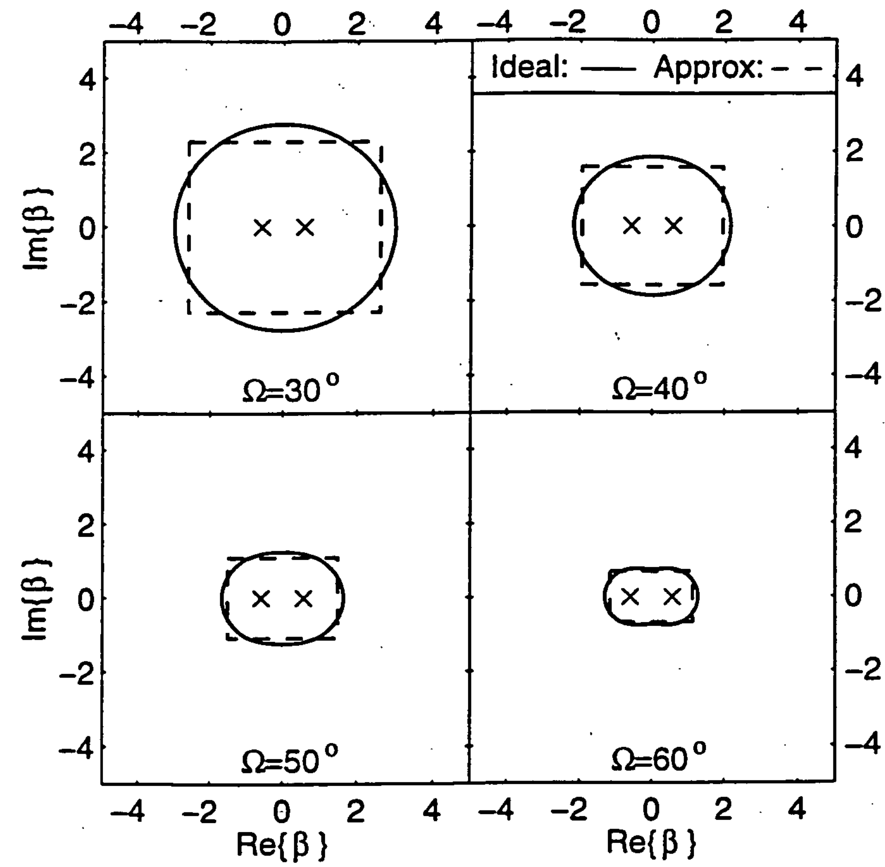


FIG. 8



: Contours (——) and approximated contours (- - -) for M=3 and $\Omega(\beta_i)$ = 30°, 40°, 50°, and 60°, with $\beta_i^{\perp}$ also indicated (x).

FIG. 9

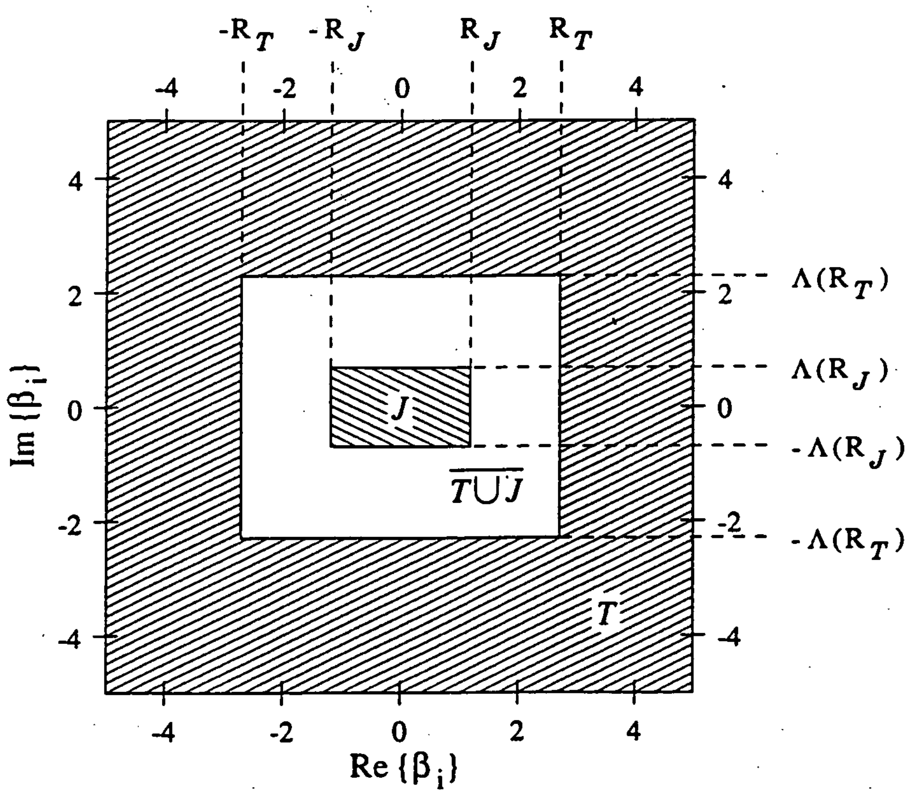
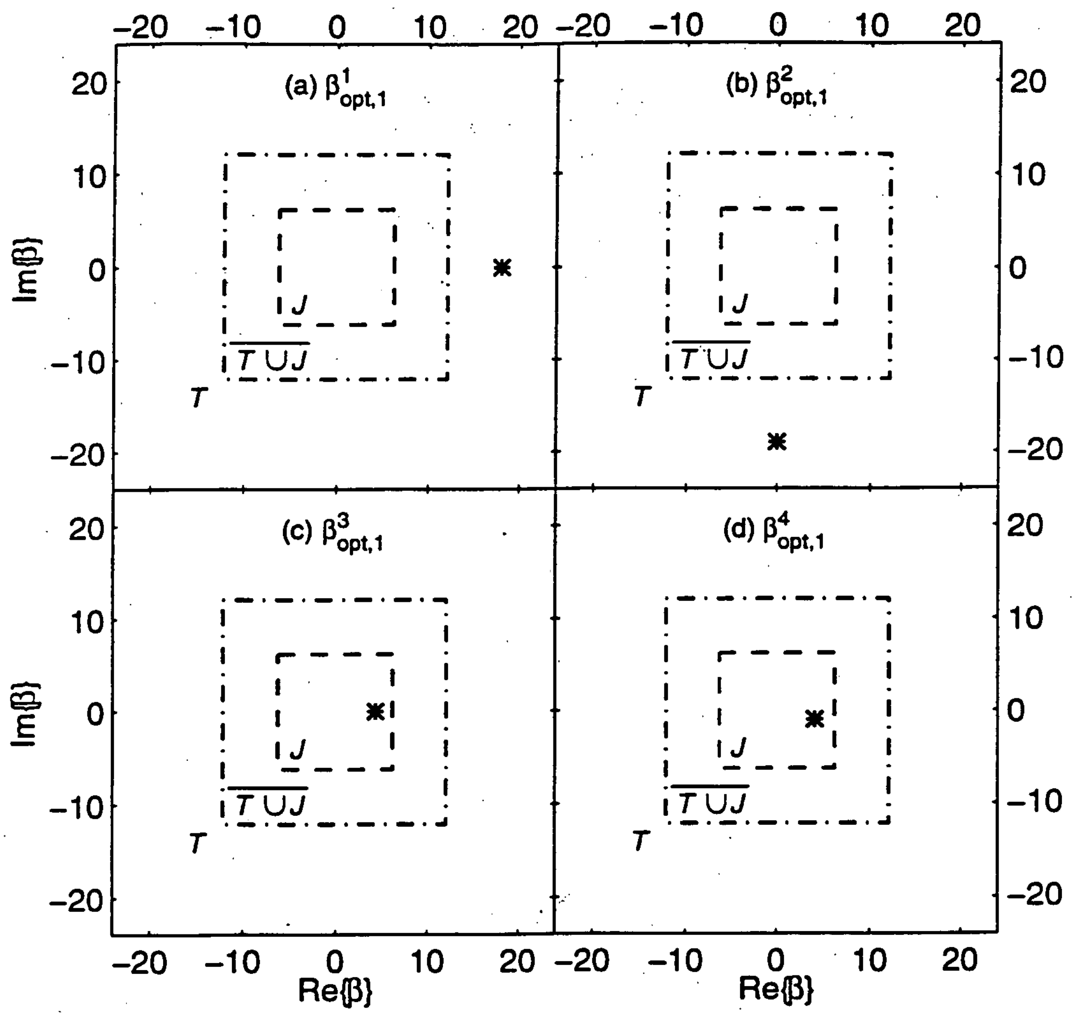


Diagram showing the three modified LENS parameter classification regions for an M=3 element array as according to Equation 3.23 when $R_{\mathcal{T}}$ and $R_{\mathcal{J}}$ are chosen to reflect the $\Omega_{\mathcal{T}} = 30°$ and $\Omega_{\mathcal{J}} = 60°$ contours, respectively.

FIG. 10



Four example values of $\beta_{opt,1}$ for a two-element array along with the corresponding LENS robustness regions.

FIG. 11

$\underline{X}$

$\left(\begin{array}{c}\text{Steered so that}\\ \underline{H}_{\text{target}} = \underline{1}\end{array}\right)$

↓

Step 1: Solve for $\underline{\beta}_{opt}$ [Eq. 3.8]

$$\underline{\beta}_{opt} = \arg\min_{\underline{\beta}} \underline{W}_{\text{LENS},M}(\underline{\beta})^H R_{XX} \underline{W}_{\text{LENS},M}(\underline{\beta})$$

↓

Step 2: Make $\underline{\beta}_{opt}$ robust [Eqs. 3.30, 3.29]

$$\beta_{rob,i} = \bar{f}[\beta_{opt,i}]\beta_{opt,i}, \text{ where}$$

$$\bar{f}[\beta_{opt,i}] = \begin{cases} 0 & \beta_{opt,i} \in \mathcal{T} \\ 1 & \beta_{opt,i} \in \mathcal{J} \\ f[\beta_{opt,i}] & \beta_{opt,i} \in \overline{\mathcal{T} \cup \mathcal{J}} \end{cases}$$

$$f[\beta_{opt,i}] = \min\left[\frac{|Re\{\beta_{opt,i}\}| - R_{\mathcal{J}}}{R_{\mathcal{T}} - R_{\mathcal{J}}}, \frac{|Im\{\beta_{opt,i}\}| - \Lambda(R_{\mathcal{J}})}{\Lambda(R_{\mathcal{T}}) - \Lambda(R_{\mathcal{J}})}\right]$$

↓

Step 3: Form weight vector [Eq. 3.1]

$$\underline{W} = \underline{W}_{\text{LENS},M}(\underline{\beta}_{rob})$$

↓

$\underline{W}$ for beamforming

FIG. 12

$\underline{\beta}_{\zeta,opt}$

Step 1: Transform to $\underline{\beta}_{\mathrm{ns},opt}$ [Eq. 5.4]

$$\beta_{\mathrm{ns},opt,i} = j\frac{(\beta_{\zeta,opt,i}+j)+\zeta(\beta_{\zeta,opt,i}+j)}{(\beta_{\zeta,opt,i}+j)-\zeta(\beta_{\zeta,opt,i}+j)}$$

Step 2: Standard LENS Robustness Restriction [Eqs. 3.30, 3.29]

$$\beta_{\mathrm{ns},rob,i} = \bar{f}[\beta_{\mathrm{ns},opt,i}]\beta_{opt,i}, \text{ where}$$

$$\bar{f}[\beta_{\mathrm{ns},opt,i}] = \begin{cases} 0 & \beta_{\mathrm{ns},opt,i} \in \mathcal{T} \\ 1 & \beta_{opt,i} \in \mathcal{J} \\ f[\beta_{\mathrm{ns},opt,i}] & \beta_{\mathrm{ns},opt,i} \in \overline{\mathcal{T} \cup \mathcal{J}} \end{cases}$$

$$f[\beta_{\mathrm{ns},opt,i}] = \min\left[\frac{|Re\{\beta_{\mathrm{ns},opt,i}\}|-R_{\mathcal{J}}}{R_{\mathcal{T}}-R_{\mathcal{J}}}, \frac{|Im\{\beta_{\mathrm{ns},opt,i}\}|-\Lambda(R_{\mathcal{J}})}{\Lambda(R_{\mathcal{T}})-\Lambda(R_{\mathcal{J}})}\right]$$

Step 3: Transform to $\underline{\beta}_{\zeta,rob}$ [Eq. 5.4]

$$\beta_{\zeta,rob,i} = j\frac{\zeta(\beta_{\mathrm{ns},rob,i}+j)+(\beta_{\mathrm{ns},rob,i}+j)}{\zeta(\beta_{\mathrm{ns},rob,i}+j)-(\beta_{\mathrm{ns},rob,i}+j)}$$

$\underline{\beta}_{\zeta,rob}$

FIG. 13

$\underline{X}$

$\left(\begin{array}{c}\text{Steered so that}\\ \underline{H}_{\text{target}} = \underline{1}\end{array}\right)$

↓

Step 1: Skew Input [Eq. 5.1]

$$\underline{X}_\zeta = Z\underline{X}$$

↓

Step 2: Solve for $\underline{\beta}_{\zeta,opt}$ [Eq. 3.8]

$$\underline{\beta}_{\zeta,opt} = \arg \min_{\underline{\beta}\in \mathbf{C}^{M-1}} \underline{W}_{\text{LENS},M}(\underline{\beta})^H R_{X_\zeta X_\zeta} \underline{W}_{\text{LENS},M}(\underline{\beta})$$

↓

Step 3: Make $\underline{\beta}_{\zeta,opt}$ robust

↓

Step 4: Form weight vector [Eq. 3.1]

$$\underline{W} = \underline{W}_{\text{LENS},M}(\underline{\beta}_{\zeta,rob})$$

↓

$\underline{W}$ for beamforming

FIG. 14

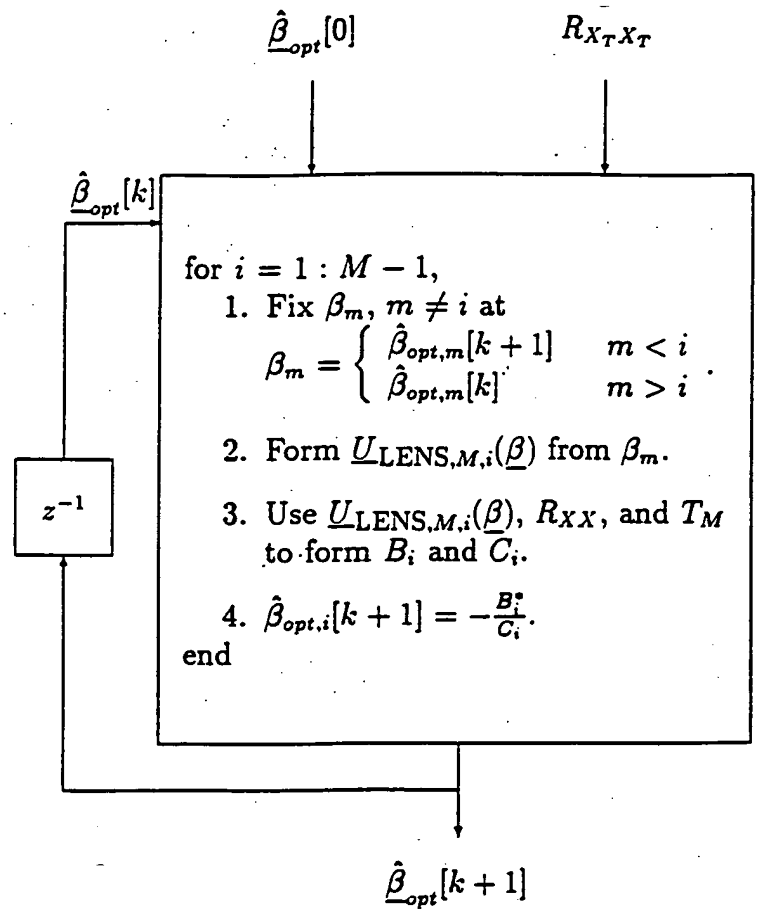
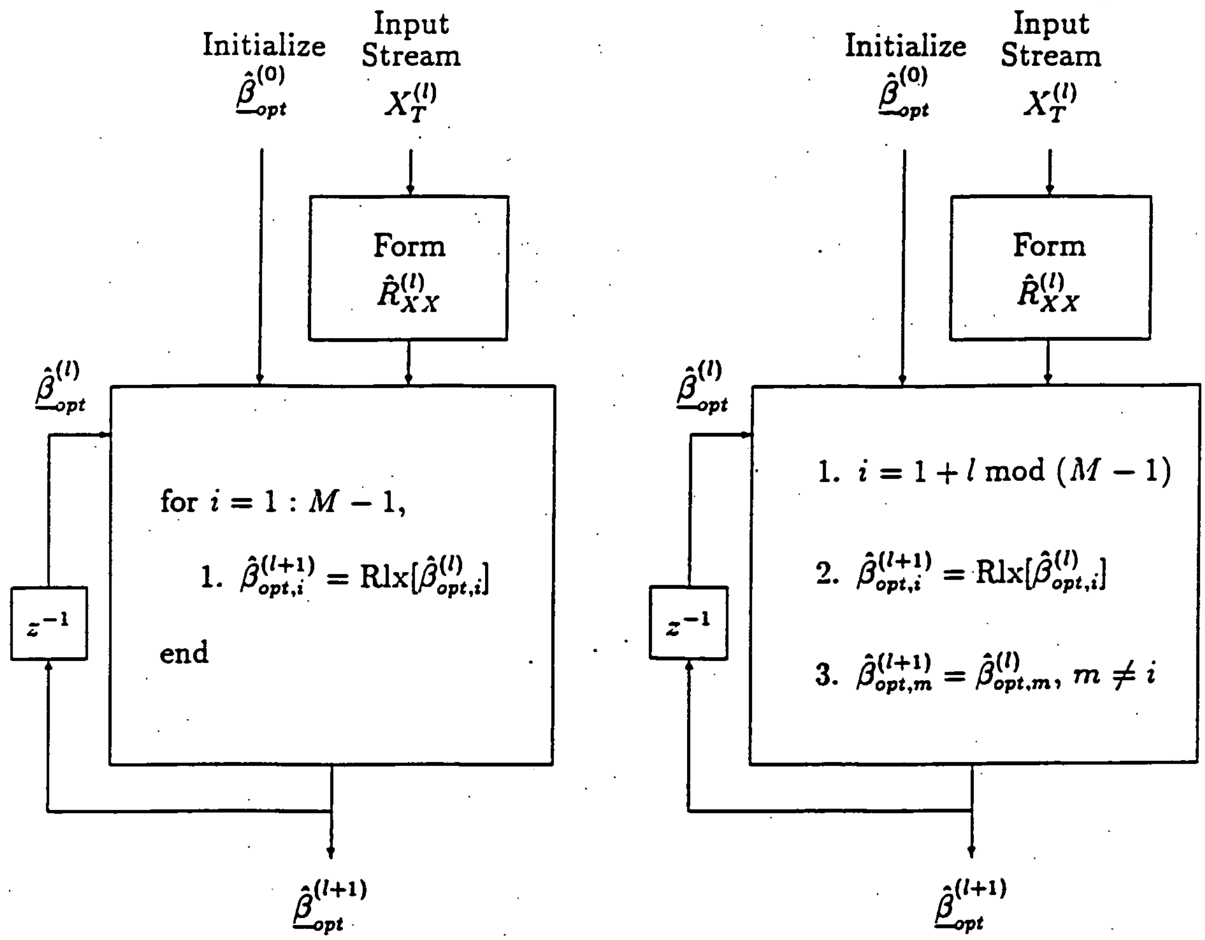

$\underline{\hat{\beta}}_{opt}[0]$
$R_{X_T X_T}$
$\underline{\hat{\beta}}_{opt}[k]$
z^{-1}
for $i = 1 : M-1$,
1. Fix β_m, $m \neq i$ at
$\beta_m = \begin{cases} \hat{\beta}_{opt,m}[k+1] & m < i \\ \hat{\beta}_{opt,m}[k] & m > i \end{cases}$.
2. Form $\underline{U}_{\mathrm{LENS},M,i}(\underline{\beta})$ from β_m.
3. Use $\underline{U}_{\mathrm{LENS},M,i}(\underline{\beta})$, R_{XX}, and T_M to form B_i and C_i.
4. $\hat{\beta}_{opt,i}[k+1] = -\frac{B_i^*}{C_i}$.
end
$\underline{\hat{\beta}}_{opt}[k+1]$

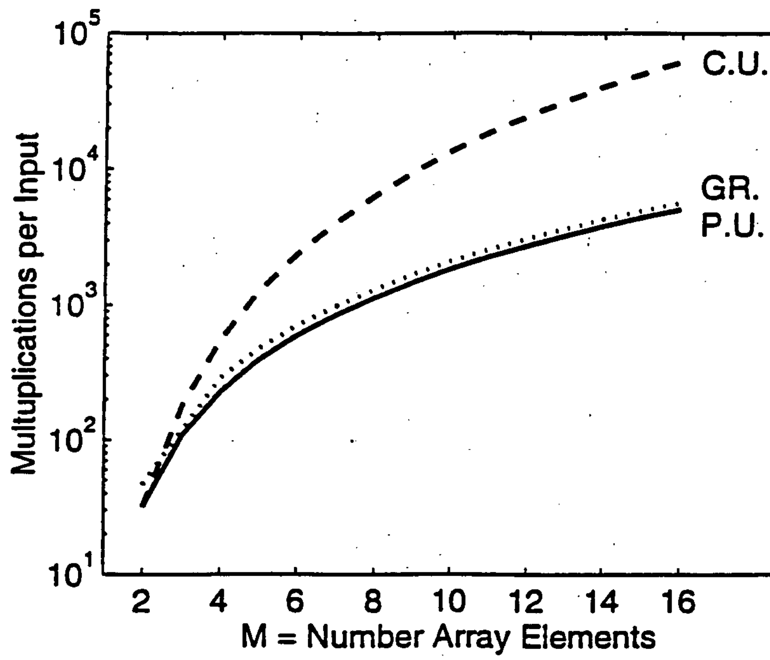
FIG. 15



(a) Complete Update

(b) Partial Update

FIG. 16



10^5
10^4
10^3
10^2
10^1
Multuplications per Input
C.U.
GR.
P.U.
2
4
6
8
10
12
14
16
M = Number Array Elements

FIG. 17